Application of the Translated-SWE Algorithm to Echo Reduction of Spherical Near-Field Measurements with Undersampling

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Abstract—In spherical Near Field (NF) measurements post-processing techniques based on spatial filtering have been presented as promising tools for the mitigation of echoes or stray signals deriving from the surrounding environment. The spatial filtering is very efficient in measurement scenarios with a stationary Antenna Under Test (AUT). Whenever the AUT is rotating, in order to increase the effectiveness of the echo reduction, the antenna needs to be displaced outside the center of rotation. Unfortunately, the measurement of the AUT in an offset configuration requires the acquisition of a number of samples higher respect to the onset configuration. An innovative spherical NF/FF transformation algorithm for offset measurements based on a Translated Spherical Wave Expansion (TSWE) has been recently proposed. In this paper, we investigate by experiment the echo reduction properties of offset AUT measurements using TSWE.

Index Terms—Spherical Wave Expansion, Spherical Near Field Measurement, Downsampling, Offset.

I. INTRODUCTION

In Near Field (NF) antenna measurement scenarios different echo reduction techniques [1-6] could be applied to enhance the accuracy of measurements, performed at lower frequencies and/or in outdoor environments as well as to reduce the impact of towers and supporting structures.

A very promising echo reduction technique is based on the so-called spatial or modal filtering. Such technique projects the measured field over a set of orthogonal and complete set of basis functions [1-2], [7, 8] and applies a filtering on the order of basis function defined according to the AUT physical dimensions [1-2]. In [5-6] it has been shown that, when the measurement is performed rotating the AUT and keeping the probe fixed, the effectiveness of the spatial/modal filtering can be sensibly improved locating the AUT in an offset position with respect to the center of rotation. Unfortunately, the measurement of the AUT in an offset configuration requires the acquisition of a number of samples that is higher respect to the onset configuration. For example, in spherical NF geometry the minimum number of NF samples needed to correctly represent the AUT depends on the radius of the minimum sphere \((R_{\text{MIN}})\) centered in the origin of the coordinate system and fully enclosing the AUT [8]. If a displacement of the antenna with respect to the center of rotation is applied, a higher number of sampling points are needed to represent the antenna with the same accuracy of the corresponding onset measurement with the drawback of an increased testing time (especially with single-probe systems).

An innovative spherical NF/FF transformation algorithm for offset measurements (Translated Spherical Wave Expansion - TSWE) has been recently proposed [9]. The TSWE is based on the modification of the Spherical Wave Functions, which are properly redefined in a new reference system located on the AUT rather than on the center of the measurement sphere (as typically done) allowing to drastically reduce the number of samples with a consequent reduction of the testing time.

II. TRANSLATED-SWE ALGORITHM

The innovative NF/FF transformation algorithm involved in this work (in the following called Translated-SWE, TSWE) is intended to be applied to offset AUT configurations measured with a strongly reduced number of samples. The optimum goal would be to perform the scan with a number of samples corresponding to the onset configuration.

With reference to the measurement layout shown in Fig. 1 the description of the TSWE algorithm is reported in this section. Let us assume a spherical NF measurement with a measurement sphere radius \((R_{\text{MEAS}})\) and a corresponding Cartesian reference system \((x, y, z)\) located in the center of the measurement sphere (see Fig. 1). An AUT is assumed to be located in an offset configuration with respect to the \((x, y, z)\) Cartesian reference system (in this case along the positive z-axis). The AUT minimum sphere defined in \((x, y, z)\) is illustrated by the red circle (see Fig. 1) and its radius is \((R_{\text{MIN,OFFSET}})\).

With these assumptions the SWE is typically performed involving the following Transmission formula [8]

\[
W(A, \chi, \theta, \varphi) = 0.5 \sum_{\sigma \mu \nu} Q_{\text{SN}}^{(3)} e^{j m \varphi} s_{\mu}^{(3)}(\theta) e^{j m \chi} c_{\sigma \mu \nu}^{(3)}(k A) R_{\sigma \mu \nu}^{p}
\]

which expresses the complex signal received by a probe \((w(A, \chi, \theta, \varphi))\) of known coefficients \((R_{\sigma \mu \nu}^{p})\) as a function of
the probes coordinates and orientation, when an AUT, described by its own spherical wave coefficient \( Q^{\text{(3)}}_{\text{smn}} \), is transmitting. The symbols \( d_{\nu}^{\mu m}(\theta) \) and \( C_{\nu}^{\mu m}(kA) \) are respectively, rotation and translation operators that, together with the two complex exponentials \( e^{j\mu\nu} \) and \( e^{j\nu\mu} \), are used to build the spherical wave expansion functions at each measurement point/orientation (assuming the \((x, y, z)\) Cartesian coordinate system). Once the spherical functions are built, the transmission formula can be solved by mean of the AUT coefficients involving the procedure described in [8]. It is remarked that, if the sampling is performed using a constant increment along both scanning axes \((\theta, \phi)\) a double FFT can be applied to compute the SWE in a very efficient way.

![Fig. 1. Illustration of an offset spherical NF measurement layout.](image)

Therefore, for the offset configuration the only drawback is the large number of samples that must be acquired in order to include in the minimum sphere the offset displacement. A larger number of modes must be computed in order to represent the AUT (which in fact has a more rapidly varying phase). In particular, as proved in [8], the number of n-modes needed to describe the antenna is

\[
N = kR_{\text{min}} + 10
\]

where in this case \( R_{\text{min}} = R_{\text{OFFSET}}\).

Let us now define the radius of the minimum sphere enclosing the antenna centered on the antenna itself. The onset minimum sphere \((R_{\text{min,ONSET}})\) is illustrated by the green circle drawn in Fig. 1. Since \((R_{\text{min,ONSET}})\) is smaller than \((R_{\text{min,OFFSET}})\), less modes are needed to represent the antenna, as indicated in (2). It is thus convenient to consider an onset minimum sphere that can be easily described defining a second coordinate system located on the AUT, the Cartesian coordinate system \((x', y', z')\) shown in Fig. 1. It turns out that the antenna offset in the Cartesian system \((x, y, z)\) is instead onset in the Cartesian system \((x', y', z')\). The spherical wave functions can thus be recomputed referring them to \((x', y', z')\). Based on the above considerations, the spherical wave functions involved in the TSWE algorithm are obtained from (1) where the roto-translation operations are performed in \((x', y', z')\).

It should be noted that, a reference system not centered in the center of the measurement sphere implies a non-equally spaced measured samples (unless a proper sampling scheme is adopted [10]). For example, if the offset is along the \(z\)-axis and the sampling over measurement sphere is performed with the typical uniform angular increments, the \(\theta\)-axis relative to \((x', y', z')\) will be non-equally spaced. Instead, if the offset is along \(x\)- and/or \(y\)-axis, both the \(\theta\)- and \(\phi\)-axes relative to the \((x', y', z')\) will be non-equally spaced. The consequence is that, the FFT cannot be applied to the non-equally spaced axis and the problem must be solved involving a matrix inversion which, as well know, is computationally less efficient.

**III. ECHO REDUCTION WITH OFFSET MEASUREMENTS**

Let us consider a spherical NF measurements performed rotating the AUT and keeping the probe fixed. The echoes are generated by a scattering object located outside the measurement sphere (for example on the side wall).

![Fig. 2. Measurement with rotating AUT, fixed Probe and reflective plate on the side wall. The AUT is onset.](image)

![Fig. 3. Measurement with rotating AUT, fixed Probe and reflective plate on the side wall. The AUT is offset.](image)
order modes in the corresponding spherical modal spectrum. In other words, the spherical modal spectrum of the AUT and the echoes are overlapped without any chance to filter out the unwanted contributions.

Instead, when the AUT is displaced out of the center of rotation of the measurement system (offset position), as depicted in Fig. 3, the length of the different reflected paths is different. In such situation, the AUT and echo signals interfere with different phases at each i-position of the AUT generating a “fast” ripple that is added to the wanted signal. Such ripple can be isolated in the spherical wave domain after mathematically translating the origin of the measurement system back to the AUT. With such measurement set-up and data processing, higher order modes associated to the echoes can be generated and thus filtered out taking into account the minimum sphere of the AUT ($R_{\text{min,ONSET}}$).

Unfortunately, in order to apply the above described procedure, a higher number of measurement samples are required to perform the offset measurement and ensure the same accuracy of an onset measurement, with the draw back of an increased testing time. With the TSWE transformation the new reference system is located directly on the offset AUT and the modal filtering could be directly applied at ($R_{\text{min,OFFSET}}$) on a down-sampled data set.

IV. VALIDATION

In order to validate the echo reduction properties of the TSWE algorithm described in the previous section the MVI SH2000 dual-ridge wideband horn has been taken into account.

Measurements have been performed in the Italian office of MVG sited in Pomezia (Rome), using a robotic arm system [11], already involved in other activities [12]. Such robot can be programmed so that it can perform different scanning schemes (e.g. planar, spherical). For this validation campaign, hemispherical NF measurements have been performed placing the AUT on the robotic arm and the probe on a tower located in front of the robot as illustrated in Fig. 4.

The alignment of the AUT and probe positioners is provided by sliding the probe tower in proximity of the robot and mating the AUT and probe interfaces with a precision mechanical adaptor.

The SH2000 maximum dimensions are 105x61x105 mm resulting in a $R_{\text{min,ONSET}}$ of approximately 81 mm (considering the coordinate system on the barycenter of the antenna). The chosen frequency for the test is 12GHz. According to (2) we thus have $N_{\text{ONSET}} = 31$ which correspond to a sampling step of approximately 5.8 deg.

The spherical NF setup used to measure the SH2000 is illustrated in Fig. 4. A scattering plate of size 0.5 m x 0.5 m (20λ @ 12GHz) has been placed outside the measurement sphere at approximately 90 cm from AUT and from the probe. The plate is intended to generate a specular reflections similar to those schematized in Fig. 2 and Fig. 3.

A stand-off of approximately 140 mm length has been used to mount the AUT on the robotic arm (see Fig. 4). Onset measurements with and without the scattering plate have been performed allowing the robotic arm to rotate around the barycenter of the AUT (center of the measurement sphere coincident with AUT barycenter). The onset measurement without the scattering plate has been considered as the reference.

![Measurement validation setup.](image)

An offset measurement has been performed moving the center of rotation of the robotic arm at the bottom of the stand-off where the antenna is mounted, generating an offset of 200 mm (or 8λ @ 12GHz) along positive z-axis. The obtained radius of the offset minimum sphere ($R_{\text{min,OFFSET}}$) is approximately 281 mm. According to (2) the number of modes needed to represent the AUT in this offset configuration would be $N_{\text{OFFSET}} = 81$, which correspond to a sampling step of approximately 2.2 deg.

Both onset and offset measurement have been performed on half a sphere with the same sampling: 4 degrees along Theta and 10 degrees along Phi. It should be noted that, with this acquisition step, the onset measurement is slightly oversampled of a factor 1.45 while the offset measurement is down-sampled of a factor 1.8.

A fourth offset measurement with the scattering plate has been performed with a sampling along Theta associated to size of the offset AUT minimum sphere ($R_{\text{min,OFFSET}}$): 2 degrees along Theta and 10 degrees along Phi. The purpose of this measurement is to compare the echo reduction properties of the TSWE algorithm to the processing that would be applied to an offset measurement of an AUT in an echoic environment using the standard sampling step.

All measurements have performed with the MVI DOEW6000 first order probe. Due to the relative low directivity of such a probe, the probe correction has been neglected during the standard and TSWE NF/FF transformations.

The comparison of directivity patterns along three cuts, Phi=0° (E-plane), Phi=45° (Diagonal-plane) and Phi=90° (H-plane) are shown in Fig. 5, Fig. 6 and Fig. 7 respectively. The blue curve is the pattern obtained applying the standard NF/FF transform on the onset data set without the reflecting plate (Reference).

The red curve is the pattern of the onset data with the reflecting plate obtained applying the standard NF/FF transform (Onset with Plate). A modal filtering based on the
AUT minimum sphere ($R_{\text{min,ONSET}}$) has been applied to such data but, as expected, it is unable to remove the effect of the scattering of the plate (a pattern distortion is visible on the left side for the Phi=0°, Phi=45° and Phi=90° cuts). In fact, as explained in the previous section, the AUT and echoes contributions are overlapped in the spherical wave spectrum resulting in a poor effectiveness of the modal filtering.

The black curve is the pattern obtained from offset measurement with plate performed with standard sampling (Offset with Plate (step=2°)). Before applying the modal filtering, a mathematical back-translation of the AUT to the origin of the coordinate system has been applied to the data. This operation, combined with the offset, allows to generate higher order spherical modes associated to the echoes. Those echoes have been removed by applying a modal filtering based on the AUT minimum sphere ($R_{\text{min,ONSET}}$). As can be seen the agreement with the Reference is very good.

Finally, the green curve is the pattern obtained applying the NF/FF transformation based on TSWE algorithm on the offset down-sampled dataset with the reflecting plate (Offset with Plate (step=4°)). As can be seen such patterns are in very good agreement with the Reference and with the corresponding echo reduced pattern obtained from the measurement performed with the standard sampling. As in the latter, the interference generated by the offset AUT and the reflected signals creates a rapidly varying ripple. Such a ripple is in this case intrinsically filtered out by the TSWE algorithm which automatically translates the origin of the reference system on the AUT and allows to filter spherical wave modes laying outside the AUT minimum sphere ($R_{\text{min,ONSET}}$).

It should be noted that the ripple on the two sides of the patterns is caused by the truncation of spherical scanning surface (in fact, due to mechanical constraint of the robotic arm, only the forward hemisphere has been measured).

In order to quantify the agreement among the different patterns the following metric called equivalent error level (EEL) has been considered

$$e_i(\theta, \phi) = \left| \frac{E(\theta, \phi) - \tilde{E}(\theta, \phi)}{\tilde{E}(\theta, \phi)} \right| \frac{\tilde{E}(\theta, \phi)}{E_{\text{EEL}}}$$

(3)

where

- $\tilde{E}(\theta, \phi)$ is the reconstructed pattern,
- $E(\theta, \phi)$ is the reference pattern

The comparison among different measurements based on the EEL is reported in Table 1. The reported EEL values have been obtained averaging the output of (3) among any ($\theta, \phi$) coordinate of the patterns.

Fig. 5. Directivity comparison @ Phi = 0° (E-Plane).

Fig. 6. Directivity comparison @ Phi = 45° (Diagonal-Plane).

Fig. 7. Directivity comparison @ Phi = 90° (H-Plane).
As expected the worst agreement is obtained by the comparison of the Reference pattern with the onset pattern with echoes (first row). An improvement of EEL of more than 10 dB is achievable by applying the echo reduction techniques on either offset dataset with standard sampling (second row) or with down-sampling (third row). The agreement between these two echo reduction approaches is excellent (fourth row). This means that the same effectiveness in the mitigation of the errors coming from the presence of echoes is obtainable combining offset measurements and modal filtering and applying the TSWE algorithm. The latter allows a significant reduction of the number of samples.

V. CONCLUSION

The TSWE algorithm has been applied to the reduction of the echo pollution in an offset measurement. The TSWE algorithm allow to significantly reduce the number of samples that would be needed in a standard offset measurement scenario.

The echo reduction properties of the TSWE have been investigated by comparative measurements performed in a controlled echoic environment using a reflecting plate. The SATIMO SH2000 wideband dual-ridge horn have been considered as antenna measurement application. The validation campaign has highlighted the echo reduction properties of the of TSWE algorithm. In fact, it has been shown that the same mitigation of errors coming from the presence of echoes obtainable combining offset measurements and modal filtering can be achieved with the TSWE algorithm allowing a significant reduction of the number of samples.

REFERENCES


