# ACCURATE INFINITE GROUNDPLANE ANTENNA MEASUREMENTS 

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#### Abstract

The accurate measurement of the infinite ground plane antenna patterns are needed in different applications as discussed in [1-12]. The comprehensive performance of a general antenna in a complex environment including interaction can be evaluated fast and accurately using ray tracing techniques [1,2]. This approach requires a reliable representation of the local source behaviour either through measurements or simulation. A good source approximation for this method is the infinite ground plane pattern assuming a perfectly conducting plane. The infinite ground plane condition can be achieved easily in simulation using fullwave computational tools but is very difficult to measure on a general antenna due to the finite dimensions of the measurement systems.

Different measurements and post processing approaches have been investigated in the past to determine the infinite ground plane pattern of a general antenna. Spherical mode truncation/filtering have been used as means to eliminate edge diffraction from finite ground plane measurements. This method suffers from the dependence on the selection of filtering parameters as discussed in [3]. Time-gating can give some information about the isolated antenna pattern in most directions as discussed in [4-6] but is not completely general and require special equipment and setup for the measurement. Other approaches to eliminate the edge diffraction by special design of the ground plane shape have also been pursued as discussed in [7-10].

This paper introduces a simple formulation to accurately determine the infinite ground plane pattern of any antenna from measurements on a small finite ground plane. The theory of the method is presented and its accuracy and suitability demonstrated with measured examples.


Keywords: Spherical near field, infinite ground plane, antenna measurements

## 1. INTRODUCTION

In spherical near field measurement systems the measured antenna field is described completely by the spherical wave spectrum determined by a near field to farfield transformation [11]. To recover the spherical wave spectrum of the finite ground plane source as if it was placed on an infinite ground plane the field scattered by the finite ground plane edges has to be removed.
In this paper we demonstrate that the technique proposed in $[12,13]$ can be extended to measurements on general sources on limited ground planes. This approach is based on the application of the image theorem on a completely general source placed on a finite size ground plane and is independent of the source position and orientation.
The novel formulation to determine the infinite ground plane antenna pattern from finite ground plane antenna measurements is based on a spherical wave expansion of the measured pattern. It is particularly suited for spherical near field systems although easily extended to any measurement system providing full sphere data.


Figure 1: Example of antenna measurement on a small circular ground plane in a StarLab system.

## 2. THEORY

The antenna consist of a small finite ground plane over which a radiating source S (SFGP) is placed as shown in Figure 2. The finite ground plane is assumed to be infinitesimally thin perfect electric conductor laying on the plane $\mathrm{z}=0$ of a Cartesian reference system ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) with unit vectors ( $\mathbf{x}, \mathbf{y}, \mathbf{z}$ ). It is assumed that the full 3D radiated farfield of the antennas can be measured with a suitable equipment [13-14].


Figure 2: Source on a finite size ground plane (SFGP).
The source $S$ represented by means of the vector current densities $\mathbf{J}$ and $\mathbf{M}$ (electric and magnetic sources respectively) is placed at a given distance from the finite ground plane in the half space $\mathrm{z}>0$. The electromagnetic field vectors $(\mathrm{E}, \mathrm{H})$ are solutions to Maxwell's equations with sources J and M and perfect electric conductor boundary conditions on the finite ground plane.
The electric current density $\mathrm{J}_{\mathrm{s}}=\mathbf{z} \times \mathrm{H}$ induced on the surface of the finite ground plane is parallel to the plane $z=0$. The image field of $\mathrm{E}, \mathrm{H}$ with respect to the plane $\mathrm{z}=0$ is $E^{\prime}, H^{\prime}$ is also a solution to Maxwell's equations in the whole space with coordinate reference system ( $x^{\prime}, y^{\prime}, z^{\prime}$ ) given by the relation $x^{\prime}=x, y^{\prime}=y$ and $z^{\prime}=-z$.
Superimposing the fields $(E, H)$ and ( $\left.E^{\prime}, H^{\prime}\right)$ we obtain a new field $\mathrm{E}_{1}=\mathrm{E}+\mathrm{E}^{\prime}$ and $\mathrm{H}_{1}=\mathrm{H}+\mathrm{H}^{\prime}$ which is a solution to maxwell's equations in the whole space with sources ( J , $\mathrm{M})$ and ( $\left.\mathrm{J}^{\prime}, \mathrm{M}^{\prime}\right)$. $\left(\mathrm{E}_{1}, \mathrm{H}_{1}\right)$ has particular symmetries with respect to the plane $\mathrm{z}=0$ and satisfies the electric wall boundary condition on that plane as $\mathbf{z} \times \mathrm{E}_{1}(\mathrm{x}, \mathrm{y}, 0)=0$ and $\mathbf{z}$ $x_{1} H_{1}(x, y, 0)=0$. Therefore, in the half space $z>0\left(E_{1}, H_{1}\right)$ can be considered a solution to Maxwell's equations with source J and M in front of an infinite perfect electric conductor plane placed at $\mathrm{z}=0$.
This demonstrates that the image theorem permits the fields scattered by the edges of an infinitesimally thin finite ground plane to be removed independently from the position, orientation and kind of radiating source. The image $E$ ' is obtained by overturning the measured data with respect to the plane $\mathrm{z}=0$.

## 3. APPLICATION TO SPHERICAL NEAR FIELD ANTENNA MEASUREMENTS

In a spherical near-field measurement system all electromagnetic field sources are inside a spherical surface of radius $r_{0}$ while the field is measured at points of a sphere of radius $r>r_{0}$ contained in a source-free space [13-14]. The measured field can be described by means of an expansion using spherical waves. The equations for the electric field components are expressed in terms of TE and TM spherical wave functions and a set of spherical wave coefficients $\left\{a_{m m}^{1}, a_{m m}^{2}\right\}$.

$$
\begin{aligned}
& E_{r}(\theta, \phi)=-\sum_{n=0}^{N} \sum_{m=-n}^{n} e^{j m \phi}\left\{\frac{n(n+1)}{k r} P_{n}^{|m|}(\cos \theta) a_{n, m}^{2} n_{n}^{(2)}(k r)\right\} \\
& E_{\theta}(\theta, \phi)=-\sum_{n=0}^{N} \sum_{m=-n}^{n} e^{j m \phi}\left\{\frac{j m}{\sin \theta} P_{n}^{|m|}(\cos \theta) a_{n, m}^{1} h_{n}^{(2)}(k r)+\frac{\partial}{\partial \theta}\left[P_{n}^{|m|}(\cos \theta)\right] a_{n, m}^{2} \frac{1}{k r} \frac{\partial}{\partial r}\left[r r_{n}^{(2)}(k r)\right]\right\} \\
& E_{\phi}(\theta, \phi)=\sum_{n=0}^{N} \sum_{m=-n}^{n} e^{j m \phi}\left\{\frac{\partial}{\partial \theta}\left[P_{n}^{|m|}(\cos \theta)\right] a_{n, m}^{1} h_{n}^{(2)}(k r)-\frac{j m}{\sin \theta} P_{n}^{|m|}(\cos \theta) a_{n, m}^{2} \frac{1}{k r} \frac{\partial}{\partial r}\left[r h_{n}^{(2)}(k r)\right]\right\}
\end{aligned}
$$

As discussed in the previous section the edge diffracted contributions from the field radiated by a source above a perfectly conductor finite ground plane is eliminated by adding the image field in the backward hemisphere to the field in the forward hemisphere with an appropriate change of sign.

If the origin of the spherical system is placed on the plane including the finite size ground plane (i.e. $\mathrm{z}=0$ ) the image of the field can be obtained overturning measured data with respect to $\theta=\pi / 2, \theta$ being the polar angle from the positive z -axis, while $r$ is the radial distance from the reference origin and $\varphi$ the azimuth angle from the positive x -axis. The superimposed electric field (corrected field) has components ( $\mathrm{E}_{\mathrm{r}}^{\mathrm{c}}, \mathrm{E}_{\theta}^{\mathrm{c}}, \mathrm{E}_{\varphi}^{\mathrm{c}}$ ) in which ( $\mathrm{E}_{\mathrm{r}}, \mathrm{E}_{\theta}, \mathrm{E}_{\varphi}$ ) are the components of the measured electric field are given by:

$$
\begin{aligned}
& E_{\theta}^{C}(\theta, \phi)=E_{\theta}(\theta, \phi)+E_{\theta}(\pi-\theta, \phi) \\
& E_{\phi}^{C}(\theta, \phi)=E_{\phi}(\theta, \phi)-E_{\phi}(\pi-\theta, \phi) \\
& E_{r}^{C}(\theta, \phi)=E_{r}(\theta, \phi)-E_{r}(\pi-\theta, \phi)
\end{aligned}
$$

Exploiting the symmetry of the Legendre functions this procedure can be applied directly to the spherical wave expansion coefficients of the measured finite ground plane radiated field. This leads to a new set of spherical wave coefficients which give the corrected field above the ground plane and its mirrored image below.

$$
\begin{aligned}
& E_{r}(\theta, \phi)=-\sum_{n=0}^{N} \sum_{m=-n}^{n} e^{j m \phi}\left\{\frac{n(n+1)}{k r} P_{n}^{|m|}(\cos \theta) c_{n, m}^{2} h_{n}^{(2)}(k r)\right\} \\
& E_{\theta}(\theta, \phi)=-\sum_{n=0}^{N} \sum_{m=-n}^{n} e^{j m \phi}\left\{\frac{j m}{\sin \theta} P_{n}^{|m|}(\cos \theta) c_{n, m}^{1} h_{n}^{(2)}(k r)+\frac{\partial}{\partial \theta}\left[P_{n}^{|m|}(\cos \theta)\right] c_{n, m}^{2} \frac{1}{k r} \frac{\partial}{\partial r}\left[r h_{n}^{(2)}(k r)\right]\right\} \\
& E_{\phi}(\theta, \phi)=\sum_{n=0}^{N} \sum_{m=-n}^{n} e^{j m \phi}\left\{\frac{\partial}{\partial \theta}\left[P_{n}^{|m|}(\cos \theta)\right] c_{n, m}^{1} h_{n}^{(2)}(k r)-\frac{j m}{\sin \theta} P_{n}^{|m|}(\cos \theta) c_{n, m}^{2} \frac{1}{k r} \frac{\partial}{\partial r}\left[r h_{n}^{(2)}(k r)\right]\right\}
\end{aligned}
$$

The new set of spherical wave coefficients are defined by the following equations:

$$
\begin{aligned}
& { }^{\theta} a_{n, m}^{1}= \begin{cases}2 a_{n, m}^{1} & \text { if } m+n \text { is even } \\
0 & \text { if } m+n \text { is odd }\end{cases} \\
& { }^{\theta} a_{n, m}^{2}= \begin{cases}0 & \text { if } m+n \text { is even } \\
2 a_{n, m}^{2} & \text { if } m+n \text { is odd }\end{cases} \\
& { }^{\phi} a_{n, m}^{1}= \begin{cases}2 a_{n, m}^{1} & \text { if } m+n \text { is even } \\
0 & \text { if } m+n \text { is odd }\end{cases} \\
& { }^{\phi} a_{n, m}^{2}= \begin{cases}0 & \text { if } m+n \text { is even } \\
2 a_{n, m}^{2} & \text { if } m+n \text { is odd }\end{cases} \\
& c_{n, m}^{1}={ }^{\theta} a_{n, m}^{1}={ }^{\phi} a_{n, m}^{1} \\
& c_{n, m}^{2}={ }^{\theta} a_{n, m}^{2}={ }^{\phi} a_{n, m}^{2}={ }^{r} a_{n, m}^{2}
\end{aligned}
$$

A more comprehensive description of the derivation of these above equations the method and algorithms are reported in [12].

The new coefficients represent the infinite ground plane field in the upper hemisphere and a symmetrical field in the lower hemisphere that should be set to zero in the case of a true infinite ground plane.

## 3. ERROR SOURCES AND APPLICATION RANGE

Using a ground plane with a finite thickness we also have to consider the induced electric currents on the physical thickness of the finite ground plane as shown in Figure 3. These currents have a component parallel to the normal vector of the ground plane that does not vanish by using the image theorem. Therefore it is not possible to eliminate their contribution to the final field. The effect of these currents depends on the thickness and the size of the ground plane. In the case of a small ground plane the antenna couples strongly with the perimeter sides making the perimeter current stronger than that of a large ground plane. In practical applications, however, the contribution produced by perimeter currents can be made small using a ground plane sufficiently thin and large with respect to the wavelength.


Figure 3: Induced currents on a thick finite size ground plane. The edge currents does not vanish by using the image theorem.

The limitations inherent to the proposed method are the following:

1) Interaction between the antenna and ground plane is not mitigated by the method. The size of the finite ground plane should be chosen so that any antenna/edge interaction is limited.
2) The edge currents induced on the finite ground plane are not removed by the image method. The thickness of the ground plane should be minimized.
3 ) If the antenna extend below the ground plane like in the case of a horn antenna the induced currents on this part of the structure will not be removed by the method. The size of the ground plane should be sufficient to ensure that this interaction is limited.
3) The method is only valid if the measurement is performed with the upper side of the finite ground plane exactly in $\mathrm{z}=0$ in the reference coordinate system.

In practice the method works well if the ground plane dimensions are such that the size is sufficiently large and the thickness minimized. This condition is fairly easy to achieve in any normal size measurements system in most applicable frequency ranges.

## 4. VALIDATION BY EXPERIMENTS

The method was validated by experiment. Since the true infinite ground plane pattern is unavailable due to the finite size of the measurement systems the method was tested with antennas mounted on different size ground planes. A monocone antenna was used a source antenna mounted on two different size ground plane as shown in Figure 4.


## Figure 4: Monocone antenna on two different ground planes. Left: $\mathrm{D}=0.3 \mathrm{~m}$. Right: $\mathrm{D}=\mathbf{0 . 6 m}$.

The measured antenna patterns at 5.28 GHz are shown in Figure 5. At this frequency the diameters of the two ground planes are roughly $5 \lambda$ and $10 \lambda$. The measurements shown that in both cases a strong finite size ground plane effect in terms of ripple is present. The radiation pattern from the monocone antenna on an infinite ground plane should be similar to the pattern of an electric dipole.


Figure 5: Measured far field radiation pattern of mono cone antenna on finite size ground plane. $D=0.3 \mathrm{~m} / 0.6 \mathrm{~m}$ @ 5.28 GHz ( $5 \lambda$ and $10 \lambda$ roughly).

The resulting infinite ground plane pattern using the proposed method are shown in Figure 6. The approximately $5 \lambda$ diameter ground plane case still show some ripple in the radiation pattern but the expected behaviour is easily recognised in the general shape. The approximately $10 \lambda$ diameter ground plane case show the expected dipole type pattern with hardly any residual ripple.


Figure 6: Application of proposed method to measured mono cone antenna on finite size ground plane. $\mathrm{D}=0.3 \mathrm{~m} / 0.6 \mathrm{~m}$ @ 5.28 GHz ( $5 \lambda$ and $10 \lambda$ roughly).

The method was applied to a truncated circular waveguide antenna as shown in Figure 7. The truncated waveguide operates in single linear polarisation and was mounted on two cases of finite circular ground planes of sizes $D=0.3 \mathrm{~m} \& 0.6 \mathrm{~m}$ corresponding to roughly $\mathrm{D}=4.4 \lambda$ \& 8.8 $@ 4.38 \mathrm{GHz}$.

The measured antenna patterns at 4.38 GHz are shown in Figure 8. The measurements shown that in both cases a strong finite size ground plane effect in terms of ripple is present particularly in the $\varphi=0^{\circ}$ cut in which the interaction with the edge is also the strongest.


Figure 7: Truncated circular waveguide with finite size ground plane. $D=0.3 \mathrm{~m} / 0.6 \mathrm{~m}$ @ 4.38 GHz ( $4.4 \lambda$ and 8.8 $\lambda$ roughly).


Figure 8: Measured far field pattern $\left(E_{\theta}, E_{\varphi}\right)$ of truncated circular waveguide on a finite size ground plane. $D=0.3 \mathrm{~m} / 0.6 \mathrm{~m}$ @ 4.38 GHz ( $4.4 \lambda$ and $8.8 \lambda$ ). Top: $\varphi=\mathbf{0}^{\circ}$, bottom: $\varphi=\mathbf{9 0}^{\circ}$.

The resulting infinite ground plane patterns using the proposed method are shown in Figure 9. The approximately $4.4 \lambda$ diameter ground plane case still show a minimum of residual ripple in the most critical $\varphi=0^{\circ}$ cut radiation pattern but the expected behaviour is easily recognised in the general shape. The approximately $8.8 \lambda$ diameter ground plane case show the expected radiation pattern from a circular aperture in an infinite ground plane with hardly any residual ripple.


Figure 9: Application of proposed method to measured truncated circular waveguide on a finite size ground plane. $D=0.3 \mathrm{~m} / 0.6 \mathrm{~m}$ @ 4.38 GHz ( $4.4 \lambda$ and 8.8 $\lambda$ ). Top: $\varphi=0^{\circ}$, Bottom: $\varphi=90^{\circ}$.

## 7. SUMMARY

A novel formulation to determine the infinite ground plane antenna pattern from a general finite ground plane antenna measurements has been presented. The method is based on a spherical wave expansion of the measured pattern. It is particularly suited for spherical near field systems although easily extended to any measurement system providing full sphere data.

Exploiting the symmetry of the Legendre functions this procedure can be applied directly to the spherical wave expansion coefficients of the measured finite ground plane radiated field. This leads to a new set of spherical wave coefficients which give the corrected field above the ground plane and its mirrored image below.

The method has been validated by measurements on different test antennas on different size ground plane. The results demonstrates that the method can be applied to any general antenna type in any position and orientation with respect to a reasonably sized ground plane and fully confirm the expected functionality of the method.

## 8. REFERENCES

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