MV Echo

Application Note

# Spherical wave expansion / modal filtering

**Within the MV-ECHO, the measured field is expanded over a set of orthogonal spherical wave modes - Spherical Wave Expansion (SWE) [4]. Such an expansion is then combined with a modal filtering based on the knowledge of the physical extent of the AUT.**

SWE is a common and well established mathematical tool used in the Spherical Near-field-to-Far-field transformation. In fact, in such a processing, the measured spherical near-field is projected (expanded) over the orthogonal Spherical Wave Functions (or Spherical Wave Modes) computing a corresponding set of complex coefficients - Spherical Wave Coefficients (SWC) or Spherical Wave Spectrum. The SWE of a generic field *E(r)* is reported in the below equation:

$$E(r)= \frac{k}{\sqrt{\eta}} \sum_{s=1}^{2} \sum_{n=1}^{\infty} \sum_{m=-n}^{\eta} Q^{(3)}_{smn} F^{(3)}_{smn}(r)$$

Where $F^{(3)}_{smn}(r)$ are the orthogonal basis functions and $Q^{(3)}_{smn}$ are the complex SWC forming the Spherical Wave Spectrum. Since the Spherical Wave modes $F^{(3)}_{smn}(r)$ are functions of the spatial variable ($r$, $\theta$, $\varphi$) the computed spectrum together with the basis function evaluated at $r \to \infty$ are used in order to estimate the far-field of the AUT.
What is important is that SWC have strong filtering capabilities and thus they can be used in an intelligent way in order to improve the accuracy of the performed measurement.
The maximum index of the significant spherical wave coefficients (SWC) in the SWE is determined by the radius $r_0$ of the smallest sphere centred at the origin and enclosing the AUT (minimum sphere) [4-5]. As a consequence, the truncation index $N$ can be determined on the basis of the knowledge of the AUT size.
More specifically, it is known that for an AUT corresponding to a minimum sphere of radius $r_0$ the coefficients of the spherical modes with index $n>N$ (with $N=kr_0+n_1$, where $n_1$ is a number depending on the size of the AUT and on the position of the measurement sphere) are expected to be negligible.
On the other hand, echoes arising outside the AUT

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minimum sphere are highly oscillating, and are associated to modes of higher order with respect to the AUT modal distribution ($n>N$), that can be easily filtered out.
It is worth noting that, since the physical extent of the AUT plays a key role in this type of filtering, any mathematical operation aimed at minimizing the AUT minimum sphere within the reference coordinate system, increases the modal separation between the AUT and the echoes enforcing the effectiveness of the method.
An overview of algorithm implemented within the MV-ECHO is depicted in Figure 1. As can be seen, the AUT minimum sphere can be optimized by MV-ECHO before computing the SWC (i.e. Coordinate System Modification).

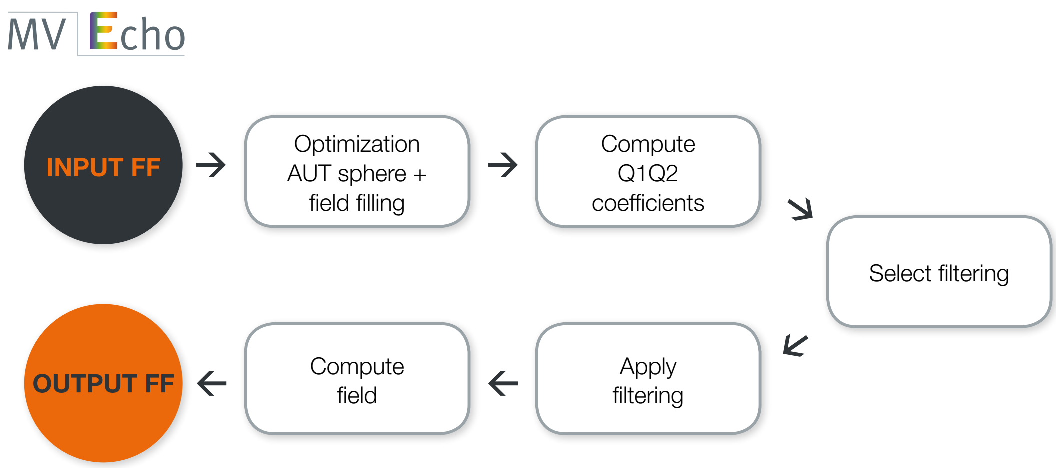


**Figure 1.** Layout of MV-ECHO algorithm.

# Spatial filtering capabilities of the SWE

In order to understand the spatial filtering capabilities of the SWE, let us considered a generic source located in different offset positions with respect to the origin of the coordinate system as illustrated in the upper part of Figure 2. In this example, a horn antenna has been simulated in four different positions (x = 0, x = 3λ, x = 6λ and x = 9λ). For each configuration, the corresponding minimum spheres are illustrated by the red circles, while the corresponding Spherical Wave Spectrum is reported below. It can be seen that the more the AUT is offset from the origin of the coordinate system, the higher is the modal content associated to that particular configuration. In other words, there is a direct link between physical extent of a certain source and mode indices (an elementary radiating element located at $P_0$ in the space domain is associated to a mode index with is equal to $N = k^*P_0$).

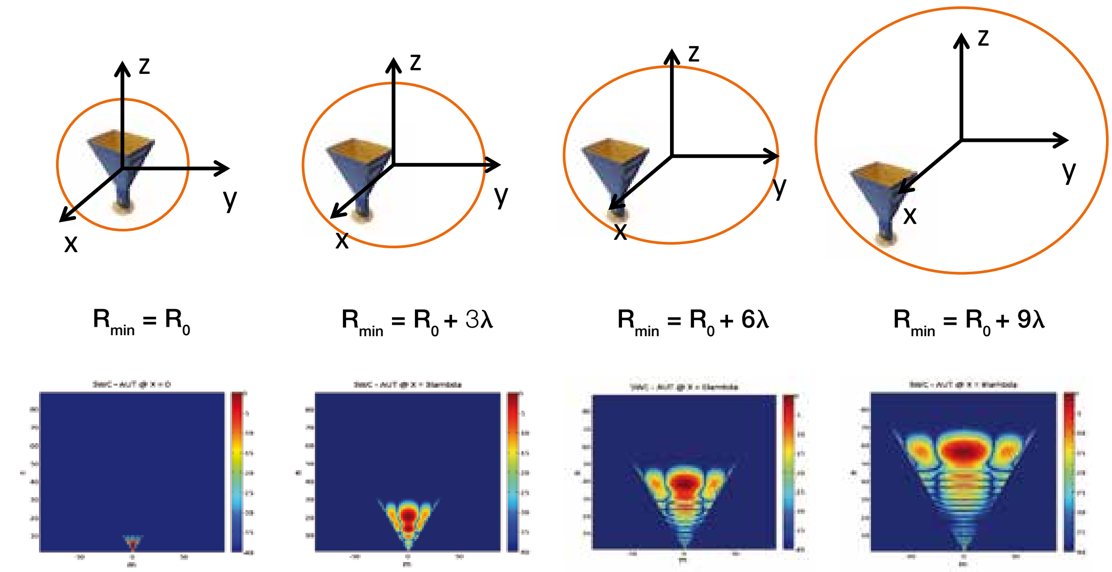


**Figure 2.** Generic source located in different offset position and the corresponding SWE.

Figure 3 shows the so-called Pn-curve (square summation of spherical wave spectrum along m-indices) associated to the four source configurations. As expected, the peak of each curve corresponds to N = k*$R_{min}$ where $R_{min}$ is the radius of the minimum spheres shown in Figure 2.

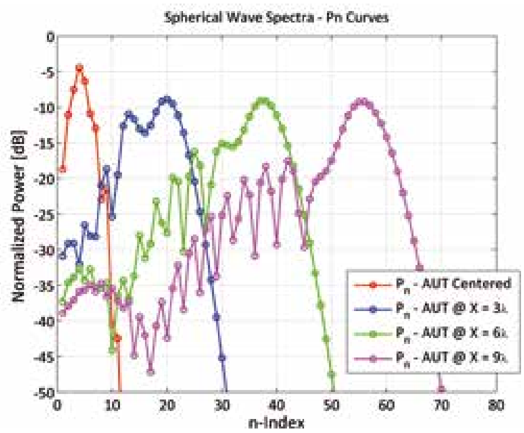


**Figure 3.** Pn curve associated to the four source configurations.

From the above considerations, it becomes clear that if an AUT or a generic DUT is located in the origin of the coordinate system and an unwanted echo-source is located outside the DUT minimum sphere, the two contributions will be well separated in the spherical wave spectrum. More specifically the DUT will be associated to lower order modes (low pass behaviour) while the echo will be associated to higher modes that can be easily filtered out by forcing them to zero. This situation is schematized in Figure 4.

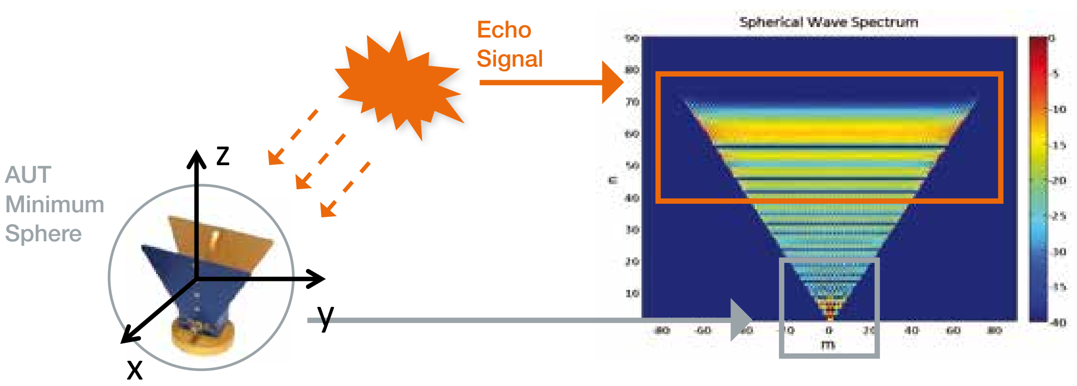


**Figure 4.** Illustration of spatial filtering capabilities of the SWE.

**For more information about MV-Echo, please visit our website:**
http://www.satimo.com/software/MV-Echo

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